

Universality of fluctuations in the Kardar-Parisi-Zhang class in high dimensions and its upper critical dimension

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We show that the theoretical machinery developed for the Kardar-Parisi-Zhang (KPZ) class in low dimensions are obeyed by the restricted solid-on-solid (RSOS) model for substrates with dimensions up to $d = 6$. Analyzing different restriction conditions, we show that height distributions of the interface are universal for all investigated dimensions. It means that fluctuations are not negligible and, consequently, the system is still below the upper critical dimension at $d = 6$. The extrapolation of the data to dimensions $d \geq 7$ predicts that the upper critical dimension of the KPZ class is infinite.

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Interface dynamics in nature is mostly a non-equilibrium process [1] and the Kardar-Parisi-Zhang (KPZ) universality class introduced by the equation [2]

$$\frac{\partial h(x, t)}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \xi, \quad (1)$$

where ξ is a white noise of zero mean and amplitude \sqrt{D} , is certainly one of the most relevant problems in non-equilibrium interface science [3, 4].

Much is known about KPZ class in $d = 1 + 1$ including exact solutions [5], experimental realizations [6], and fine-tuning properties as universal schema for finite-time corrections [6–9] and for the crossover to the stationary regime [10]. The compilation of all results gave rise to the extended KPZ ansatz, whose main ideas were formerly introduced by Krug *et al.* [11], for the interface evolution in the non-stationary regime where the height at each surface point evolves as

$$h = v_\infty t + s_\lambda (\Gamma t)^\beta \chi + \eta + \dots, \quad (2)$$

where $s_\lambda = \text{sgn}(\lambda)$, β is the growth exponent, and χ is an stochastic variable, whose universal distribution depends only on the growth geometry [12]. The spatial correlations, which also depend on the growth geometry, are also known and given by the so-called Airy processes [3]. The parameters v_∞ , Γ and η are non-universal, being the last one responsible by a shift in the distribution of the scaled height,

$$q = \frac{h - v_\infty t}{s_\lambda (\Gamma t)^\beta}, \quad (3)$$

in relation to the asymptotic distributions χ . Except for the very specific case where $\langle \eta \rangle = 0$ [7], the shift vanishes as $\langle q \rangle - \langle \chi \rangle \sim t^{-\beta}$.

Simulations of several models that, in principle, belong to the KPZ class have shown that the ansatz given by Eq. (2) can be extended to $d = 2 + 1$ with universal and geometry-dependent stochastic fluctuations [13–16]. Despite of a possible fragility of the KPZ equation to non-local perturbation discussed in Ref. [17], the universality

for flat growth in $d = 2 + 1$ was recently observed in semiconductor [18] and organic [19] films.

The KPZ class in dimensions $d \geq 3$ is still an open problem with basic unanswered questions. In particular, the upper critical dimension d_u above which fluctuations become irrelevant, independently of the strength of non-linearity of the KPZ equation, is a controversial unsolved problem. Several works based on mode-coupling theory [20] and field theoretical approaches [21] suggest $2.8 \lesssim d_u \leq 4$, whereas renormalization group approaches [22, 23] and simulations of KPZ models [24–29] show that KPZ upper critical dimension, if it exists, is higher than four. Particularly, in Refs. [23, 28] was suggested that $d_u = \infty$. A short but comprehensive review of the state of the art is presented in Ref. [24].

Much of these discussions on d_u were based on scaling exponents. The squared interface width, defined as the variance of the interface height profile, evolves according to the Family-Vicsek ansatz [30]

$$W^2(L, t) \equiv \langle h^2 \rangle_c = L^{2\alpha} f\left(\frac{t}{L^z}\right), \quad (4)$$

where $\langle X^n \rangle_c$ denotes the n th cumulant of X . The scaling function $f(x) = x^{2\beta}$ for $x \ll 1$ and $f(x) = \text{const}$ if $x \gg 1$. The roughness α and dynamical z exponents obey the scaling relation $\alpha + z = 2$ independent of the dimension [3], which was checked up to $5+1$ dimensions [26]. For $1 \ll t \ll L^z$, $W \sim t^\beta$ where the growth exponent is given by $\beta = \alpha/z$. For $d \geq d_u$, we have $\alpha = \beta = 0$ and $z = 2$.

In this work, we investigate the classical restricted solid-on-solid (RSOS) model proposed by Kim and Kosterlitz [31], which is widely accepted as a prototype of the KPZ universality class [24], at the light of the KPZ ansatz for interface fluctuations given by Eq. (2). In the RSOS model, particles are deposited in random positions of an initially flat substrate, represented by a d -dimensional lattice, given that the height differences among nearest neighbors are not larger than a certain integer value m , the height restriction parameter. We analyzed substrates with dimension up to $d = 6$ and

show that in all dimensions the model follows the theoretical KPZ machinery for interface fluctuations given by Eq. (2) as well as growth velocity dependence on system size and substrate slope [32]. Furthermore, the cumulant ratios and, consequently, the height distributions are universal and depend only on d for distinct height restriction parameters in all investigated dimensions. It reads as fluctuations are not negligible and, consequently, we are still below the critical dimension for $d = 6$. The distributions and corrections to the scaling are also presented.

It was shown that differences greater than unity in the height restriction parameter of the RSOS model reduce finite-size and -time corrections and, consequently, improves the scaling analysis [27]. So, we used height restriction $m = 2$ to 8, depending on dimension. The case $m = 1$ is hampered by an initial layer-by-layer growth which makes the determination of universal quantities very hard in high dimensions [27]. We considered d -dimensional substrates of sizes up to L_{max} and periodic boundary conditions. Only results for the largest ones are shown. Sizes $L_{max} = 2^9, 2^7, 2^6, 2^5$ and number of independent samples $N_s = 2500, 1000, 250, 50$ were used for dimensions $d = 3$ to 6, respectively. In all cases, typically $\sim 10^{11}$ surface points were used to compute statistics of interface distributions.

According to Eqs. (2) and (4), the growth exponent can be determined using the time dependence of $\langle h^2 \rangle_c$. Figure 1(a) shows the effective growth exponent against time for RSOS model in several dimensions. The exponents extrapolated to $t \rightarrow \infty$ are shown in Table I. The obtained values are in very good agreement with the recentest results for $d \leq 5$ [26, 27], in particular for $d = 4$, where most of the discussion about the upper critical dimension has held on. Our estimate for $d = 6$ is consistent but more accurate than former simulations presented in Ref. [29]. The interface velocity was calculated with Eq. (2): The time derivative of $\langle h \rangle$ plotted against $t^{\beta-1}$ is extrapolated via a linear regression [9]. The interface velocities against $t^{\beta-1}$ for the RSOS model with $m = 4$ are shown in Fig. 1(b). Their asymptotic values are summarized in Table I.

The shift of the distribution is investigated defining the quantities $g_1 = (\langle h \rangle_t - v_\infty)/\beta \rightarrow \Gamma^\beta \langle \chi \rangle$ and $q' = (h - v_\infty t)/(s_\lambda g_1 t^\beta)$ [9]. Figure 1(c) shows the curves $g_1(t)$ for RSOS model with $m = 4$ and the corresponding extrapolated values. Equation (2) implies that $1 - \langle q' \rangle \simeq -(s_\lambda \eta)/g_1 t^{-\beta}$, which is confirmed in Fig. 1(d). Therefore, the KPZ ansatz including the correction η , Eq. (2), is very well obeyed by RSOS model in $d = 3 - 6$ dimensions, in analogy with the lower dimensional cases [8, 9, 15]. These results simultaneously generalize the validity of the KPZ ansatz and confirm that RSOS model belongs to KPZ class in higher dimensions.

Important properties of χ can be achieved through di-

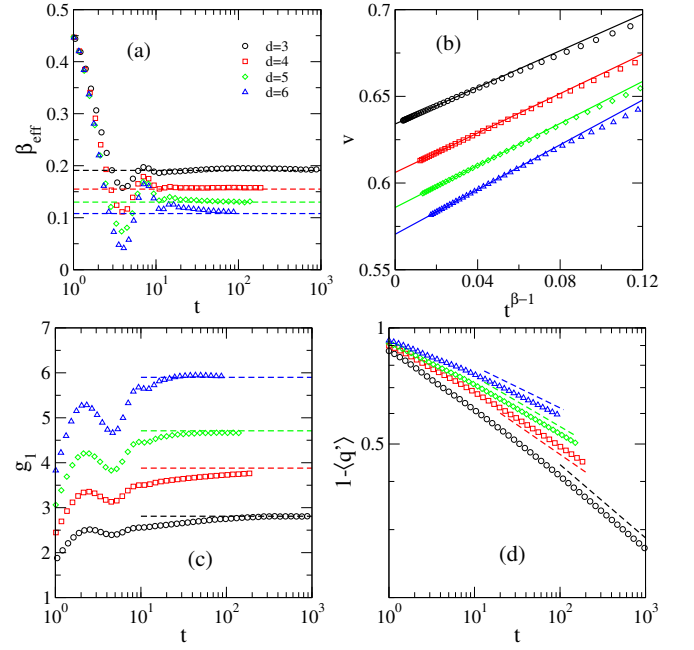


FIG. 1. (a) Effective growth exponent defined as $d \ln W / d \ln t$ for RSOS model in $d + 1$ dimensions with height restriction $m = 4$. Dashed lines are the estimates for β . (b) Curves used to determine the asymptotic interface velocity. Solid lines are linear regressions. (c) Determination of the quantity $\Gamma^\beta \langle \chi \rangle$. Horizontal lines are the extrapolated values. (d) Scaling of the shift in relation to the asymptotic distribution. Dashed lines are power laws $t^{-\beta}$ using β exponents given in Table I.

mensionless cumulant ratios. So, we define quantities $g_n = \langle h^n \rangle_c / t^{n\beta}$ for $n \geq 2$ that, according to Eq. (2), goes to $\Gamma^\beta \langle \chi^n \rangle_c$ for $t \rightarrow \infty$ [9]. Therefore, the dimensionless ratios $R = g_2/g_1^2$, $S = g_3/g_2^{3/2}$ (skewness), and $K = g_4/g_2^2$ (kurtosis) are independent of the particular model in the case of a universal χ . Figures 2(a)-(c) shows the dimensionless cumulant ratios against inverse of t , where one can see convergence to values that do not vary with substrate dimension but does not with the height restriction parameter. Extrapolated values are given in Table I.

Above d_u , where the surface is essentially flat, one expects non-universal cumulant ratios. For example, the linear Edwards-Wilkinson (EW) equation [33], which is obtained with $\lambda = 0$ in the KPZ equation, has an exact solution and its upper critical dimension is $d_u = 2$ [1]. The numerical integration [34] of the EW equation (with $\nu = 2.5$ and $D = 0.5$) results in kurtosis $K \approx -0.09, -0.18$ and -0.26 in $d = 3, 4$ and 5 , respectively. On the other hand, simulations of the random deposition with surface relaxation (RDSR) [35], a typical discrete model in the EW class, provide $K \approx 0.95, 2.7$ and 4.3 in the same dimensions. Notice that the up-down symmetry of EW systems implies in a null skewness. The kurtosis of EW equation becomes more negative with the substrate dimension because the surface is smoothing for higher di-

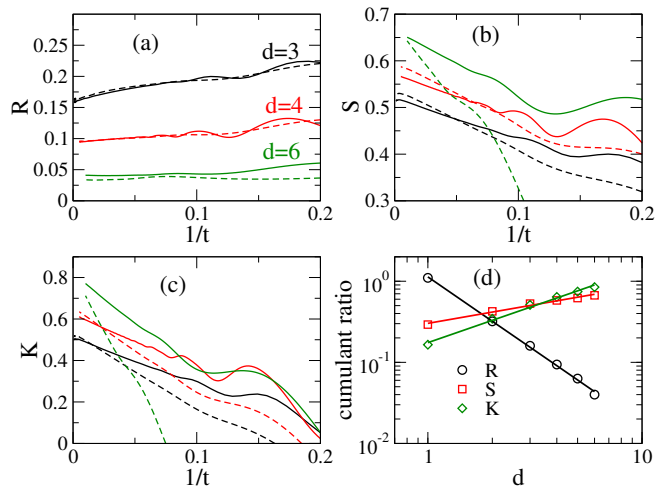


FIG. 2. Dimensionless cumulant ratios (a) $R = \langle h^2 \rangle_c / \langle h \rangle^2$, (b) $|S| = |\langle h^3 \rangle| / \langle h^2 \rangle_c^{3/2}$, and (c) $K = \langle h^4 \rangle_c / \langle h^2 \rangle_c^2$ against inverse of time for RSOS model in different dimensions indicated near the respective curves in panel (a). The height restriction parameters of Table I are shown for each dimension being the dashed lines the smaller ones. Panel (d) shows the cumulant ratios as functions of the substrate dimension. Data for $d = 1$ and 2 were taken from Refs. [9] and [15], respectively.

mensions and, therefore, the height distribution becomes narrower. Otherwise, for RDSR model the positive increasing kurtosis is due to its discrete nature where only a very few heights have non-negligible probabilities to occur.

In order to clarify the latter kurtosis dependence in discrete systems, consider a toy surface with a height distribution $P(\delta) = q$ if $\delta = -1$, $P(\delta) = 1 - p - q$ if $\delta = 0$, $P(\delta) = p$ if $\delta = +1$, and $P(\delta) = 0$ otherwise. Both parameters p and q are very small mimicking the dependence on dimension at $d > d_u$ and possibly the non-

| Model | β | v_∞ | R | S | K |
|------------------|---------|------------|-------|------|------|
| $d = 3$ | | | | | |
| RSOS ($m = 2$) | 0.189 | 0.44650 | 0.156 | 0.53 | 0.50 |
| RSOS ($m = 4$) | 0.191 | 0.6340 | 0.163 | 0.53 | 0.52 |
| $d = 4$ | | | | | |
| RSOS ($m = 2$) | 0.150 | 0.41518 | 0.093 | 0.57 | 0.63 |
| RSOS ($m = 4$) | 0.155 | 0.6059 | 0.096 | 0.59 | 0.65 |
| $d = 5$ | | | | | |
| RSOS ($m = 2$) | 0.13 | 0.39356 | 0.064 | 0.61 | 0.73 |
| RSOS ($m = 4$) | 0.13 | 0.5858 | 0.063 | 0.63 | 0.76 |
| $d = 6$ | | | | | |
| RSOS ($m = 4$) | 0.11 | 0.57055 | 0.042 | 0.66 | 0.83 |
| RSOS ($m = 8$) | 0.10 | 0.7380 | 0.037 | 0.68 | 0.86 |

TABLE I. Numerical results for RSOS models in $d + 1$ dimensions.

universal properties/parameters of the surface dynamics. One has $\langle \delta^n \rangle = p - q$ for n odd and $\langle \delta^n \rangle = p + q$ for n even. To the leading order, the cumulants are, therefore, $\langle \delta^n \rangle_c \simeq \langle \delta^n \rangle$. So, skewness and kurtosis are $S \simeq (p - q)/(p + q)^{3/2}$ and $K \simeq 1/(p + q)$, respectively, which are clearly dependent on the parameters p and q . Moreover, the smoother the interface (smaller p and q) the larger S and K . So, the independence of RSOS model on the height restriction parameter is a strong evidence that the model is still below d_u .

A smooth surface for $d \geq d_u$ also implies that the cumulant ratio R is null. Figure 2(d) shows the dependence of the cumulant ratios with dimension. Our data corroborate the discussion above: Increasing d leads to smoother surfaces and, thus, to decreasing R and increasing S and K . An approximately power-law dependence on dimensionality is found for all investigated ratios, being $R \sim d^{-1.8}$, $S \sim d^{0.46}$ and $K \sim d^{0.92}$. If these power laws hold for any dimension, the KPZ class does not have an upper critical dimension, as previously conjectured [23, 28].

The cumulant ratio analysis allows us to obtain essentially all statistics of the distribution in terms of the first cumulant [9, 15], which must be determined independently using the non-universal parameters Γ and v_∞ [6, 16]. The non-universal parameter controlling the amplitude fluctuations in KPZ ansatz can be obtained by the relation $\Gamma = |\lambda|A^{1/\alpha}$ that, apart from some dimensionless prefactor, holds for KPZ equation below critical dimension ($\alpha, \beta > 0$) [11]. The parameter λ can be determined using deposition on tilted large substrates with an overall slope s , for which a simple dependence between velocity and slope, $v = v_\infty + \frac{\lambda}{2}s^2$ is expected for the KPZ class [32]. The parameter A also has a simple relation with asymptotic velocity v_L for finite systems of size L [32], $\Delta v = v_L - v_\infty \simeq -\frac{A\lambda}{2}L^{\alpha-2}$. We used the estimates of α exponents reported in Ref. [26]. Figure 3 shows the data analysis to determine A and λ parameters for $d = 3$ and 4 . We were not able to accurately perform these plots for higher dimensions since tilting is hampered by the small substrate sizes currently attainable. The values found are shown in Table II.

The first cumulant can then be obtained using Eq. (2):

$$\langle \chi \rangle = \frac{\langle h \rangle - v_\infty t}{s_\lambda(\Gamma t)^\beta} - \frac{\langle \eta \rangle}{s_\lambda(\Gamma t)^\beta} + \dots, \quad (5)$$

where the finite-time correction $t^{-\beta}$ is explicitly used to extrapolate $\langle \chi \rangle$. Analogously, the second cumulant can be obtained using $\langle \chi^2 \rangle_c \simeq \langle h^2 \rangle_c / (\Gamma t)^{2\beta} + \dots$, in which corrections do not follow a universal scheme [6, 8, 9, 15]. Figures 4(a) and (b) show typical plots for cumulant determination. As one can notice, the extrapolation is imperative to the estimate of $\langle \chi \rangle$ from finite-time data, fact neglected in the first analysis of two-dimensional models at light on the KPZ ansatz [13]. Indeed, our estimate of $\langle \chi \rangle$ for $d = 2$ is slightly larger than the former estimate

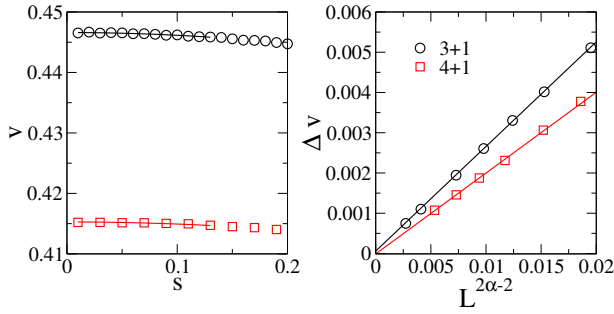


FIG. 3. Determination of non-universal parameters in RSOS model with height restriction $m = 2$ for three- and four-dimensional substrates. Lines are parabolic (left) or linear (right) regressions to determine the parameters λ and A , respectively.

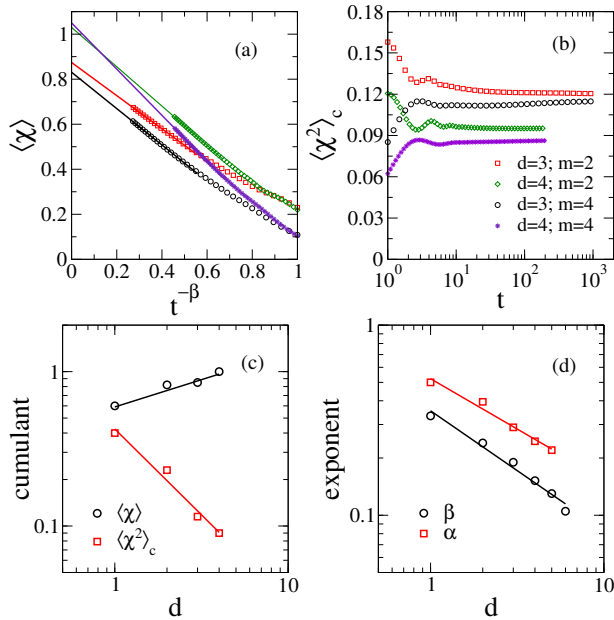


FIG. 4. Determination of (a) first and (b) second cumulant of χ . Lines are linear regressions used to extrapolate $\langle\chi\rangle$. The dependence on the substrate dimension of the universal (c) cumulants and (d) exponents. Lines are power law regressions. The roughness exponents were taken from Ref. [26].

for RSOS model [13] but completely consistent with a more refined analysis done later [16]. The asymptotic cumulants are shown in Table II. One can observe that the first cumulant gets more negative, while the variance decreases as dimension increases. Finally, possessing v_∞ , Γ and $\langle\eta\rangle$ the density probability distribution can be drawn for different dimensions, see Fig. 5. What do we observe is that distributions vary less as dimension is increased but no special hallmark can be highlighted.

Further evidences supporting the absence of an upper critical dimension are given in Figs. 4(c) and (d) where cumulants and exponents are drawn against substrate dimension. In both pictures no signature of an upper criti-

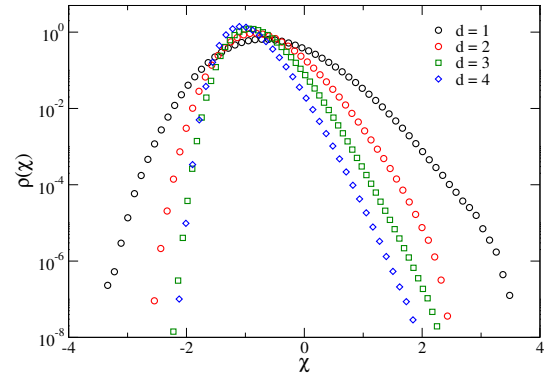


FIG. 5. Density probability distribution for RSOS model at substrates with dimension $d = 1$ to 4. The case $d = 1$ was not rescaled by the factor $2^{1/3}$ to render the Gaussian orthogonal ensemble (GOE) distribution [6].

cal behavior can be resolved. On the contrary, both scaling exponents and variance seem to slowly converge to zero roughly obeying power laws $\beta \sim d^{-0.63}$, $\alpha \sim d^{-0.53}$, and $\langle\chi^2\rangle_c \sim d^{-1.1}$. In particular, the RG analysis of Castellano *et al.* [23] foresaw a roughness exponent decaying slower than $1/d$ that is fully supported by our current analysis.

TABLE II. Estimates of non-universal parameters (A , λ , Γ) for RSOS model in $d = 1$ to 4 dimensions. Height restriction parameters are shown in brackets. The estimates of the first and second cumulant of χ are shown in last columns. Results for $d = 1$ were extracted from Ref. [8] where a factor different convention $\Gamma = |\lambda|A/2$ was used. Our results in $d = 1$ and 2 with $m = 1$ are in agreement with former reports [11, 16].

| d [m] | A | λ | Γ | $\langle\chi\rangle$ | $\langle\chi^2\rangle_c$ |
|-------------|---------|-----------|-----------|----------------------|--------------------------|
| 1 [1] | 0.81 | -0.77 | 0.51 | -0.60 | 0.40 |
| 2 [1] | 1.22(4) | -0.41(1) | 0.68(6) | -0.83(2) | 0.23(1) |
| 2 [2] | 4.5(1) | -0.121(3) | 5.5(2) | -0.82(2) | 0.23(1) |
| 3 [2] | 5.8(2) | -0.090(2) | 38(3) | -0.86(2) | 0.12(1) |
| 3 [4] | 19(2) | -0.024(2) | 600(50) | -0.82(3) | 0.11(1) |
| 4 [2] | 8(1) | -0.05(1) | 240(50) | -1.00(4) | 0.09(1) |
| 4 [4] | 25(2) | -0.015(2) | 7600(900) | -0.98(5) | 0.09(1) |

In summary, we performed extensive simulations of the RSOS model on substrates with dimension up to $d = 6$. We showed that the KPZ ansatz, given by Eq. (2) and initially conjectured for $d = 1$, holds also in dimensions $d = 3 - 6$, extending a recent generalization to $d = 2$ [8, 13]. Furthermore, the asymptotic growth velocities were shown to follow the slope and size dependence predicted by the theoretical machinery for the KPZ class [32]. The height distributions found are independent of the height restriction parameter, pointing out its universality. Our results also rule out a critical dimension $d_u \leq 6$. The extrapolations of universal quantities to higher dimensions are consistent with the absence of an

upper critical dimension. We expect that our results will motivate the analysis of curved growth in high dimensions, extending the geometry-dependent universality of χ to $d \geq 3$. Additionally, the high-dimensional analogues of the Airy processes for spatial covariance in $d = 1$ [3], which only very recently was numerically determined in $d = 2$ [19], is an interesting problem that deserves further attention.

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